New representation of the field equations – looking for a new vacuum solution for QMAG

Elvis Baraković

University of Tuzla
Department of Mathematics

19. rujna 2012.

Structure of presentation

Structure of presentation

- Metric – affine gravity (MAG)

Structure of presentation

- Metric – affine gravity (MAG)
- Quadratic metric – affine gravity (QMAG)

Structure of presentation

- Metric – affine gravity (MAG)
- Quadratic metric – affine gravity (QMAG)
- Solutions of QMAG

Structure of presentation

- Metric – affine gravity (MAG)
- Quadratic metric – affine gravity (QMAG)
- Solutions of QMAG
- PP – spaces

Structure of presentation

- Metric – affine gravity (MAG)
- Quadratic metric – affine gravity (QMAG)
- Solutions of QMAG
- PP – spaces
- Main result

Structure of presentation

- Metric – affine gravity (MAG)
- Quadratic metric – affine gravity (QMAG)
- Solutions of QMAG
- PP – spaces
- Main result
- Discussion

Metric – affine gravity

Alternative theory of gravity.

Alternative theory of gravity.

Natural generalization of Einstein's GR, which is based on a spacetime with Riemannian matric g of Lorentzian signature.

Metric – affine gravity

Alternative theory of gravity.

Natural generalization of Einstein's GR, which is based on a spacetime with Riemannian matric g of Lorentzian signature.

We consider spacetime to be a connected real 4-manifold M equipped with Lorentzian metric g and an affine connection Γ.

SPACETIME MAG=$\{M, g, \Gamma\}$

Metric – affine gravity

$$
\begin{aligned}
MAG &\Rightarrow R \neq 0 \wedge T \neq 0, \\
GR &\Rightarrow R \neq 0 \wedge T = 0.
\end{aligned}
$$

Metric – affine gravity

$$\begin{aligned} MAG &\Rightarrow R \neq 0 \wedge T \neq 0, \\ GR &\Rightarrow R \neq 0 \wedge T = 0. \end{aligned}$$

The 10 independent components of the symmetric metric tensor $g_{\mu\nu}$ and 64 connection coefficients $\Gamma^{\lambda}{}_{\mu\nu}$ are unknowns of MAG.

In QMAG, we define our action as

$$S := \int q(R) \qquad (1)$$

where $q(R)$ is a quadratic form on curvature R.

Metric – affine gravity

In QMAG, we define our action as

$$S := \int q(R) \tag{1}$$

where $q(R)$ is a quadratic form on curvature R.

The quatratic form $q(R)$ has 16 R^2 terms with 16 real coupling constants.

In QMAG, we define our action as

$$S := \int q(R) \tag{1}$$

where $q(R)$ is a quadratic form on curvature R.

The quatratic form $q(R)$ has 16 R^2 terms with 16 real coupling constants.

Why we use quadratic form?

Quadratic metric – affine gravity

The system of Euler – Lagrange equations:

$$\frac{\partial S}{\partial g} = 0, \qquad (2)$$

$$\frac{\partial S}{\partial \Gamma} = 0. \qquad (3)$$

Quadratic metric – affine gravity

The system of Euler – Lagrange equations:

$$\frac{\partial S}{\partial g} = 0, \tag{2}$$

$$\frac{\partial S}{\partial \Gamma} = 0. \tag{3}$$

Objective: To study the combined system of field equations (2) and (3) which is **system of 10+64 real nonlinear PDE with 10+64 real unknowns.**

Special case of $q(R)$ is

$$q(R) := R^{\kappa}{}_{\lambda\mu\nu} R^{\lambda}{}_{\kappa}{}^{\mu\nu}$$

so we get Yang - Mills theory for the affine connection.

Solutions of QMAG

Special case of $q(R)$ is

$$q(R) := R^{\kappa}{}_{\lambda\mu\nu} R^{\lambda}{}_{\kappa}{}^{\mu\nu}$$

so we get Yang - Mills theory for the affine connection.

Riemannian and non–Riemannain solutions.

Special case of $q(R)$ is

$$q(R) := R^{\kappa}{}_{\lambda\mu\nu} R^{\lambda}{}_{\kappa}{}^{\mu\nu}$$

so we get Yang - Mills theory for the affine connection.

Riemannian and non–Riemannain solutions.

Definition

We call a spacetime $\{M, g, \Gamma\}$ Riemannian if the connection is Levi–Civita, i.e. $\Gamma^{\lambda}{}_{\mu\nu} = \left\{ {\lambda \atop \mu\nu} \right\}$ *and non–Riemannian otherwise.*

Solutions of QMAG

Only after these variations we set the connection to be Levi–Civita and consider Riemannian solutions of the field equations.

Solutions of QMAG

Only after these variations we set the connection to be Levi–Civita and consider Riemannian solutions of the field equations.

D. Vassiliev proved that the following spacetimes

Solutions of QMAG

Only after these variations we set the connection to be Levi–Civita and consider Riemannian solutions of the field equations.

D. Vassiliev proved that the following spacetimes

- Einstein spaces ($Ric = \Lambda g$),
- pp-spaces with parallel Ricci curvature (pp–metric$+\nabla Ric = 0$), and
- Riemannian spacetimes which have zero scalar curvature and are locally a product of Einstein 2–manifolds (Levi–Civita$+\mathcal{R} = 0$),

are solutions of the system (2),(3).

Solutions of QMAG

Definition

We call a spacetime $\{M, g, \Gamma\}$ *a pseudoinstanton if the connection is metric–compatible and curvature is irreducible amd simple..*

Solutions of QMAG

Definition

We call a spacetime $\{M, g, \Gamma\}$ a pseudoinstanton if the connection is metric–compatible and curvature is irreducible amd simple..

It is the case that there are only three types od pseudoinstantons:

Solutions of QMAG

Definition

We call a spacetime $\{M, g, \Gamma\}$ *a pseudoinstanton if the connection is metric–compatible and curvature is irreducible amd simple..*

It is the case that there are only three types od pseudoinstantons:

- *scalar pseudoinstanton*: $R^{(1)} \neq 0$,
- *pseudoscalar pseudoinstanton*: $R_*^{(1)} \neq 0$,
- *Weyl pseudoinstanton*: $R^{(10)} \neq 0$.

Solutions of QMAG

Definition

We call a spacetime $\{M, g, \Gamma\}$ a pseudoinstanton if the connection is metric–compatible and curvature is irreducible amd simple..

It is the case that there are only three types od pseudoinstantons:

- *scalar pseudoinstanton*: $R^{(1)} \neq 0$,
- *pseudoscalar pseudoinstanton*: $R_*^{(1)} \neq 0$,
- *Weyl pseudoinstanton*: $R^{(10)} \neq 0$.

Theorem (Vassiliev)

A pseudoinstanton is a solution of the field equations (2), (3).

Classical pp–spaces

PP–spaces are well–known spacetimes in GR (Brinkmann,Peres).

Classical pp–spaces

PP–spaces are well–known spacetimes in GR (Brinkmann,Peres).

Meaning of the pp?

Classical pp–spaces

PP–spaces are well–known spacetimes in GR (Brinkmann,Peres).

Meaning of the pp?

A very simple formula for curvature: only trace–free Ricci and Weyl.

Classical pp–spaces

PP–spaces are well–known spacetimes in GR (Brinkmann,Peres).

Meaning of the pp?

A very simple formula for curvature: only trace–free Ricci and Weyl.

Definition

A pp–wave is a Riemannian spacetime whose metric can be written locally in the form

$$ds^2 = 2dx^0dx^3 - (dx^1)^2 - (dx^2)^2 + f(x_1, x_2, x_3)(dx^3)^2 \qquad (4)$$

in some local coordinates (x^0, x^1, x^2, x^3).

Generalized pp-waves

In a classical pp–space we consider the polarized Maxwell equation

$$* \, dA = \pm i dA. \tag{5}$$

Generalized pp-waves

In a classical pp–space we consider the polarized Maxwell equation

$$* \, dA = \pm i dA. \tag{5}$$

We seek plane wave solution of (5):

$$A = h(\varphi)m + k(\varphi)l$$

$$\varphi : M \to \mathbb{R}, \quad \varphi(x) := \int_M l \cdot dx.$$

Generalized pp-waves

In a classical pp–space we consider the polarized Maxwell equation

$$* \, dA = \pm i dA. \tag{5}$$

We seek plane wave solution of (5):

$$A = h(\varphi)m + k(\varphi)l$$

$$\varphi : M \to \mathbb{R}, \quad \varphi(x) := \int_M l \cdot dx.$$

Definition

A generalized pp-wave is a metric compatible spacetime with pp–metric and torsion

$$T := \frac{1}{2}\mathrm{Re}(A \otimes dA).$$

New representation of the field equations

We write down explicitly our field equations (2), (3) under following assumptions:

New representation of the field equations

We write down explicitly our field equations (2), (3) under following assumptions:

(*i*) our spacetime is metric compatible,

(*ii*) curvature has symmetries

$$R_{\kappa\lambda\mu\nu} = R_{\mu\nu\kappa\lambda}, \qquad \varepsilon^{\kappa\lambda\mu\nu} R_{\kappa\lambda\mu\nu} = 0,$$

(*iii*) scalar curvature is zero.

Main result

The main result is

Main result

The main result is

Lemma

Under the above assumptions $(i) - (iii)$, *the field equations (2), (3) are*

Main result

The main result is

Lemma

Under the above assumptions $(i) - (iii)$, the field equations (2), (3) are

$$0 = d_1 \mathcal{W}^{\kappa\lambda\mu\nu} Ric_{\kappa\mu} + d_3 \left(Ric^{\lambda\kappa} Ric_{\kappa}{}^{\nu} - \frac{1}{4} g^{\lambda\nu} Ric_{\kappa\mu} Ric^{\kappa\mu} \right) \quad (6)$$

New representation of the field equations

$$
\begin{aligned}
0 &= d_6 \nabla_\lambda Ric_{\kappa\mu} - d_7 \nabla_\kappa Ric_{\lambda\mu} \\
&+ d_6 \Big(Ric_\kappa{}^\eta (K_{\mu\eta\lambda} - K_{\mu\lambda\eta}) + \frac{1}{2} g_{\lambda\mu} \mathcal{W}^{\eta\zeta}{}_{\kappa\xi} (K^\xi{}_{\eta\zeta} - K^\xi{}_{\zeta\eta}) + \frac{1}{2} g_{\mu\lambda} Ric_\xi{}^\eta K^\xi{}_{\eta\kappa} \\
&\qquad + g_{\mu\lambda} Ric_\kappa{}^\eta K^\xi{}_{\xi\eta} - K^\xi{}_{\xi\lambda} Ric_{\kappa\mu} + \frac{1}{2} g_{\mu\lambda} Ric_\kappa{}^\xi (K^\eta{}_{\xi\eta} - K^\eta{}_{\eta\xi}) \Big) \\
&- d_7 \Big(Ric_\lambda{}^\eta (K_{\mu\eta\kappa} - K_{\mu\kappa\eta}) + \frac{1}{2} g_{\kappa\mu} \mathcal{W}^{\kappa\zeta}{}_{\lambda\xi} (K^\xi{}_{\eta\zeta} - K^\xi{}_{\zeta\eta}) + \frac{1}{2} g_{\mu\kappa} Ric_\xi{}^\eta K^\xi{}_{\eta\lambda} \\
&\qquad + g_{\kappa\mu} Ric_\lambda{}^\eta K^\xi{}_{\xi\eta} - K^\xi{}_{\xi\kappa} Ric_{\lambda\mu} + \frac{1}{2} g_{\mu\kappa} Ric_\lambda{}^\xi (K^\eta{}_{\xi\eta} - K^\eta{}_{\eta\xi}) \Big) \\
&+ b_{10} \Big(g_{\mu\lambda} \mathcal{W}^{\eta\zeta}{}_{\kappa\xi} (K^\xi{}_{\zeta\eta} - K^\xi{}_{\eta\zeta}) + g_{\mu\kappa} \mathcal{W}^{\eta\zeta}{}_{\lambda\xi} (K^\xi{}_{\eta\zeta} - K^\xi{}_{\zeta\eta}) \\
&\qquad + g_{\mu\lambda} Ric_\kappa{}^\xi (K^\eta{}_{\eta\xi} - K^\eta{}_{\xi\eta}) + g_{\mu\kappa} Ric_\lambda{}^\xi (K^\eta{}_{\xi\eta} - K^\eta{}_{\eta\xi}) \\
&\qquad + g_{\kappa\mu} Ric_\lambda{}^\eta K^\xi{}_{\xi\eta} - g_{\lambda\mu} Ric_\kappa{}^\eta K^\xi{}_{\xi\eta} + Ric_{\mu\kappa} K^\eta{}_{\lambda\eta} - Ric_{\mu\lambda} K^\eta{}_{\kappa\eta} \Big) \\
&+ 2b_{10} \Big(\mathcal{W}^\eta{}_{\mu\kappa\xi} (K^\xi{}_{\eta\lambda} - K^\xi{}_{\lambda\eta}) + \mathcal{W}^\eta{}_{\mu\lambda\xi} (K^\xi{}_{\kappa\eta} - K^\xi{}_{\eta\kappa}) \\
&\qquad - K_{\mu\xi\eta} \mathcal{W}^{\eta\xi}{}_{\kappa\lambda} - K^\xi{}_{\xi\eta} \mathcal{W}^\eta{}_{\mu\lambda\kappa} \Big)
\end{aligned} \tag{7}
$$

where $d_1, d_3, d_6, d_7, b_{10}$ are some real constants.

Discussion

Theorem

Generalized pp–spaces of parallel Ricci curvature are solutions of the system (6), (7).

Discussion

Theorem

Generalized pp–spaces of parallel Ricci curvature are solutions of the system (6), (7).

The proof is done by 'brute force'.

Discussion

Theorem

Generalized pp–spaces of parallel Ricci curvature are solutions of the system (6), (7).

The proof is done by 'brute force'.

Singh: *On axial vector torsion in vacuum quadratic Poincaré gauge field theory* solutions of the vacuum field equations with purely axial torsion.

Discussion

Theorem

Generalized pp–spaces of parallel Ricci curvature are solutions of the system (6), (7).

The proof is done by 'brute force'.

Singh: *On axial vector torsion in vacuum quadratic Poincaré gauge field theory* solutions of the vacuum field equations with purely axial torsion.

Conjecture

There exits a purely axial torsion waves which are solution of the field equations (2), (3).

Discussion

We are going to try to generalize pp–waves as follows

Discussion

We are going to try to generalize pp–waves as follows

Conjecture

There exists a new class of spacetimes with pp–metrics and purely axial torsion which are solution of the field equations (2), (3).

Discussion

We are going to try to generalize pp–waves as follows

Conjecture

There exists a new class of spacetimes with pp–metrics and purely axial torsion which are solution of the field equations (2), (3).

Expectations:

Discussion

We are going to try to generalize pp–waves as follows

Conjecture

There exists a new class of spacetimes with pp–metrics and purely axial torsion which are solution of the field equations (2), (3).

Expectations:

- to prove two conjectures above.

Discussion

We are going to try to generalize pp–waves as follows

Conjecture

There exists a new class of spacetimes with pp–metrics and purely axial torsion which are solution of the field equations (2), (3).

Expectations:

- to prove two conjectures above.
- to give a physical interpretation of the new solutions and compare them with existing Riemannian solutions.

Thank you! Welcome to Tuzla!