Purely Axial Torsion Waves – New Solutions of Metric-affine Gravity

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Structure of talk

- Mathematical model
- PP-waves with axial torsion
- New vacuum solutions of quadratic metric-affine gravity
- Physical interpretation

Metric-affine gravity

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An *independent* linear connection Γ distinguishes MAG from GR - g and Γ viewed as two totally independent quantities. Action is

$$S := \int q(R),$$

where $q(R)$ is a Lorentz invariant purely quadratic form on curvature.

Field equations

Euler–Lagrange equations

$$\partial S/\partial g = 0, \quad (1)$$
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The Yang–Mills action for the affine connection is a special case

$$q(R) := R^{\kappa}{}_{\lambda\mu\nu} R^{\lambda}{}_{\kappa}{}^{\mu\nu}.$$

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$$ds^2 = 2\,dx^0\,dx^3 - (dx^1)^2 - (dx^2)^2 + f(x^1, x^2, x^3)\,(dx^3)^2$$

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Well known spacetimes in GR, simple formula for curvature.

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Torsion (3) clearly purely axial.

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- Ricci curvature is

$$Ric = \frac{1}{2}(f_{11} + f_{22} - k^2)\, l \otimes l.$$

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Generalised pp-waves with purely axial torsion of parallel $\{Ric\}$ are solutions of the system of equations (1), (2).

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- Torsion and torsion generated curvature are waves traveling at the speed of light.
- Underlying pp-space can be viewed as the ‘gravitational imprint’ created by wave of some massless field.
- Mathematical model for some massless particle?

Metric-affine vs Einstein-Weyl

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Look at massless Dirac (or Weyl) action

$$S_W := 2i \int \left(\xi^a \, \sigma^\mu{}_{a\dot{b}} \, (\nabla_\mu \bar{\xi}^{\dot{b}}) \; - \; (\nabla_\mu \xi^a) \, \sigma^\mu{}_{a\dot{b}} \, \bar{\xi}^{\dot{b}} \right),$$

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In a generalised pp-space Weyl's equation takes form

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$$S_{\mathrm{EW}} := k \int \mathcal{R} + S_{\mathrm{neutrino}},$$

$$\partial S_{\mathrm{EW}} / \partial g = 0,$$

$$\partial S_{\mathrm{EW}} / \partial \xi = 0.$$

Thank You very much and welcome to Tuzla!