# Physical Interpretation of PP-waves With Axial Torsion

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A spacetime with pp-metric and torsion

$$T := *A \tag{1}$$

where $A$ is a real vector field defined by $A = k(\varphi)l$.
(*V. Pasic and E. Barakovic: "Torsion wave solutions in Yang-Mielke theory of gravity*", Advances in High Energy Physics, accepted for publication. )

The torsion $T$ is purely axial and the connection $\Gamma$ is metric compatible.

The remarkable property: the curvature of a generalised pp-wave is a sum of the curvature of the underlying classical pp-space

$$-\frac{1}{2}(l \wedge \{\nabla\}) \otimes (l \wedge \{\nabla\}) f \qquad (2)$$

and the curvature

$$\frac{1}{4}k(\varphi)^2 \mathrm{Re}\left((l \wedge m) \otimes (l \wedge \bar{m})\right) \mp \frac{1}{2}k'(\varphi) \mathrm{Im}\left((l \wedge m) \otimes (l \wedge \bar{m})\right) \qquad (3)$$

generated by a axial torsion wave traveling over the pp-space.
Ricci curvature is

$$Ric = \frac{1}{2}\left(f_{11} + f_{22} - k^2\right)(l \otimes l). \qquad (4)$$

and scalar curvature $\mathcal{R}$ is equal to zero.

Our goal: to compare the generalised pp-waves with purely axial torsion to the solutions of the classical models describing the interaction of gravitational and massless neutrino fields ( EW theory).

Our torsion and torsion generated curvature can be interpreted as waves traveling at speed of light.

The underlying classical pp-space of parallel Ricci curvature can then be viewed as the gravitational imprint created by a wave of some massless matter field.

We deal with the complexified curvature $\mathfrak{R} := r\,(l \wedge m) \otimes (l \wedge \overline{m})$, where $r := \frac{1}{4}k^2 - \frac{i}{2}k'$, hence $R_T = Re(\mathfrak{R})$. The curvature $\mathfrak{R}$ is polarized, i.e. ${}^*\mathfrak{R} = \mathfrak{R}^* = \pm \mathrm{i}\mathfrak{R}$, and it wan be written as

$$\mathfrak{R}_{\alpha\beta\gamma\delta} = \sigma_{\alpha\beta ab}\,\omega^{abcd}\,\overline{\sigma}_{\gamma\delta cd} \tag{5}$$

Resolving (5) with respect to $\omega$ yields

$$\omega = \xi \otimes \xi \otimes \xi \otimes \xi$$

where

$$\xi := r^{1/4}\,\chi \tag{6}$$

and $\chi^a = (1,\ 0)$.

The spinor (6) satisfies Weyl's (the massless Dirac) equation.

# Einstein–Weyl field equation

We consider the action as

$$S_{EW} := 2i \int \left( \xi^a \, \sigma^\mu{}_{a\dot{b}} \, (\{\nabla\}_\mu \bar{\xi}^{\dot{b}}) \;-\; (\{\nabla\}_\mu \xi^a) \, \sigma^\mu{}_{a\dot{b}} \, \bar{\xi}^{\dot{b}} \right) + K \int \mathcal{R}. \qquad (7)$$

The explicit representation of the Einstein–Weyl field equations is

$$\frac{i}{2}\left[\sigma^\nu{}_{a\dot{b}} \left( \bar{\xi}^{\dot{b}} \{\nabla\}^\mu \xi^a - \xi^a \{\nabla\}^\mu \bar{\xi}^{\dot{b}} \right) + \sigma^\mu{}_{a\dot{b}} \left( \bar{\xi}^{\dot{b}} \{\nabla\}^\nu \xi^a - \xi^a \{\nabla\}^\nu \bar{\xi}^{\dot{b}} \right)\right]$$
$$+ i \left( \xi^a \, \sigma^\eta{}_{a\dot{b}} \, (\{\nabla\}_\eta \bar{\xi}^{\dot{b}}) g^{\mu\nu} - (\{\nabla\}_\eta \xi^a) \, \sigma^\eta{}_{a\dot{b}} \, \bar{\xi}^{\dot{b}} g^{\mu\nu} \right) - K Ric^{\mu\nu} + \frac{K}{2} \mathcal{R} g^{\mu\nu} = 0, \qquad (8)$$
$$\sigma^\mu{}_{a\dot{b}} \{\nabla\}_\mu \, \xi^a = 0. \qquad (9)$$

The system (8), (9) has the solutions in the form of pp-waves.

# Comparison of metric–affine and Einstein–Weyl solutions

We wish to present a class of explicit solutions of EW field equations where the metric $g$ is in the form of a pp-metric and the spinor $\xi = r^{1/4}\xi$.

The condition that a pp-wave needs to satisfy to be a solution of Einstein–Weyl is

$$f_{11} + f_{22} = k(x^3)^2 + \frac{2i}{K}\left((r^{1/4})'\,\overline{r^{1/4}} - r^{1/4}\,(\overline{r^{1/4}})'\right), \tag{10}$$

The differences:
MA case: the generalised pp-wave solutions have parallel $\{Ric\}$ curvature
EW case: the pp-wave type solutions do not necessarily have parallel Ricci curvature.

MA case: Laplacian of $f$ can be any constant,
EW case: Laplacian of $f$ required to be a particular constant.

# Physical interpretation

The generalised pp-waves of parallel Ricci curvature are very similar to pp-type solutions of the Einstein–Weyl model.

The generalised pp-waves represent a metric-affine model for the massless neutrino.